

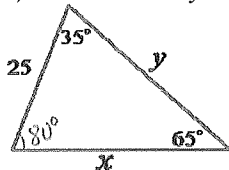
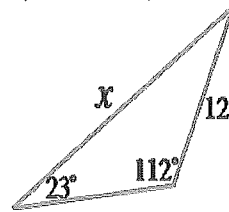
Name: Dora Li

Date: \_\_\_\_\_

**Pre-Calculus 11: Lesson 5 HW Sine Law**1. Given each equation, solve for all values of  $\theta$  where  $0 \leq \theta \leq 180^\circ$ 

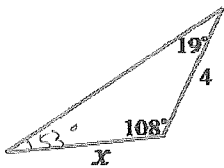
|                                                            |                                                            |                                                                    |                                                                    |
|------------------------------------------------------------|------------------------------------------------------------|--------------------------------------------------------------------|--------------------------------------------------------------------|
| a) $\sin \theta = 0.25$<br>$14.48^\circ$<br>$165.52^\circ$ | b) $\sin \theta = 0.85$<br>$58.21^\circ$<br>$121.79^\circ$ | c) $\sin \theta = \frac{\sqrt{3}}{2}$<br>$60^\circ$<br>$120^\circ$ | d) $\sin \theta = \frac{\sqrt{2}}{2}$<br>$45^\circ$<br>$135^\circ$ |
| e) $\sin \theta = 1.2$<br>no solution                      | f) $\sin \theta = -0.25$<br>no solution                    | g) $\sin \theta = 0$<br>$0^\circ, 180^\circ$                       | h) $\sin \theta = 1$<br>$90^\circ$                                 |

2. Given each triangle, find the value of any missing side or angle "x" and "y". Show all your work

|                                                                                                                                                                                                                                   |                                                                                                                                                                                |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| a) $x = 15.83$ $y = 27.17$<br><br>$\frac{x}{\sin 35^\circ} = \frac{25}{\sin 65^\circ} = \frac{y}{\sin 80^\circ}$<br>$x = 15.83$<br>$y = 27.17$ | b) $x = 28.48$<br><br>$\frac{x}{\sin 112^\circ} = \frac{12}{\sin 23^\circ}$<br>$x = 28.48$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|



c)  $x = 1.63$

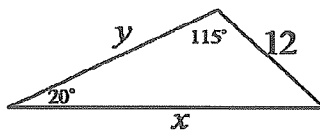


$$\frac{x}{\sin 19^\circ} = \frac{4}{\sin 53^\circ}$$

$$x = 1.63$$

d)  $x = 31.8$

$y = 24.8$



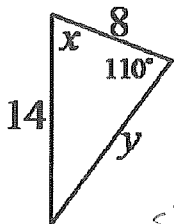
$$\frac{\sin 20^\circ}{12} = \frac{\sin 115^\circ}{x} = \frac{\sin 65^\circ}{y}$$

$$x = 31.8$$

$$y = 24.8$$

e)  $x = 37.52^\circ$

$y = 9.1$



$$\frac{\sin 110^\circ}{14} = \frac{\sin \theta}{8}$$

$$\theta = 32.48^\circ$$

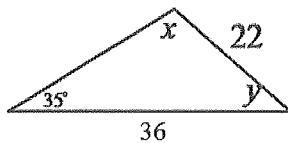
$$x = 37.52^\circ$$

$$\frac{\sin 110^\circ}{14} = \frac{\sin 37.52^\circ}{y}$$

$$y = 9.1$$

f)  $x = 69.8^\circ$

$y = 75.19^\circ$



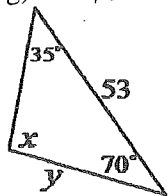
$$\frac{\sin 35^\circ}{22} = \frac{\sin x}{36}$$

$$x = 69.8^\circ$$

$$y = 75.19^\circ$$



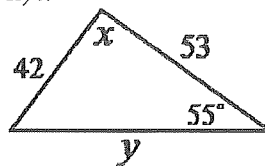
g)  $x = 75^\circ$   $y = 31.5$



$$\frac{\sin 35^\circ}{y} = \frac{\sin 75^\circ}{53}$$

$$y = 31.5$$

h)  $x =$   $y =$



$$\frac{\sin 55^\circ}{42} = \frac{\sin \theta}{53}$$

no solution.

ratio is  $> 1$

3. Given that  $a = 14\text{cm}$ ,  $b = 18\text{cm}$ , and  $\angle A = 41^\circ$ , find the area of  $\triangle ABC$ . Ambiguous Case.

case #1

$$\frac{\sin 41^\circ}{14} = \frac{\sin B}{18}$$

$$\angle B = 57.51^\circ$$

$$\angle C = 81.49^\circ$$

$$\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}(14)(18)\sin 81.49^\circ = \boxed{124.6\text{cm}^2}$$

case #2

$$\angle B = 122.49^\circ$$

$$\angle C = 16.51^\circ$$

$$\frac{1}{2}(14)(18)\sin 16.51^\circ$$

$$= \boxed{35.8\text{cm}^2}$$

4. Given that  $a = 14\text{cm}$ ,  $\angle B = 70^\circ$ , and  $\angle A = 35^\circ$ , find the area of  $\triangle ABC$ .

$$\frac{\sin 70^\circ}{b} = \frac{\sin 35^\circ}{14}$$

$$b = 23\text{cm}$$

$$\angle C = 75^\circ$$

$$\uparrow$$
  

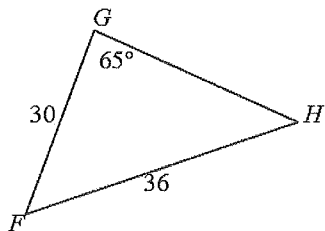
$$(180 - 70 - 35)$$

$$A = \frac{1}{2}(14)(23)\sin 75^\circ$$

$$= \boxed{155.5\text{cm}^2}$$



5. Find the value of angle "F"



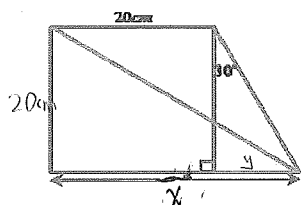
$$\frac{\sin 65^\circ}{36} = \frac{\sin H}{30}$$

no Ambiguous Case

$$\angle H = 49^\circ$$

$$\angle F = 66^\circ$$

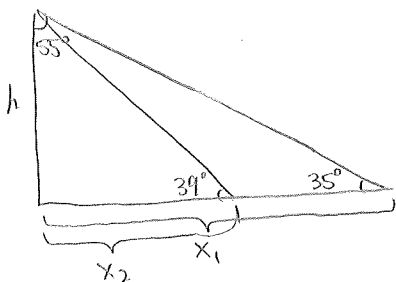
6. Calculate the length of "x" to 1 decimal place



$$\tan 30^\circ = \frac{y}{20} \quad y = 11.5 \text{ cm}$$

$$11.5 + 20 = 31.5 \text{ cm.}$$

7. An observer is looking at a mountain peak at an angle of elevation of 35 degrees. He walks 250 meters towards the mountain and the angle of elevation to the peak is 39 degrees. What is the height of the mountain?



$$\tan 35^\circ = \frac{h}{x_1}$$

$$\tan 39^\circ = \frac{h}{x_1 - 250}$$

$$0.7x_1 = 0.8(x_1 - 202.45)$$

$$202.45 = 0.1x_1$$

$$x_1 = 1840.5 \text{ m}$$

$$h = x_1 (\tan 35^\circ)$$

$$h = 1288.7 \text{ m}$$





Name: Dora Li

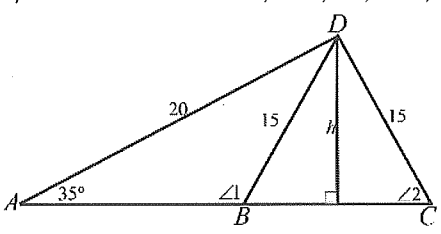
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**Pre-Calculus 11: HW 2.3b Ambiguous Case of Sine Law**1. Given each equation, solve for all values of  $\theta$  where  $0 \leq \theta \leq 360^\circ$ . Note: There are two angles!!

|                                                                                                                              |                                                                                             |                                                                                                                                                                          |
|------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| a) $\sin \theta = \frac{2}{3}$<br><br>$180 - 41.81 = 138.19^\circ$<br><br>$\theta_1 = 41.81^\circ$ $\theta_2 = 138.19^\circ$ | b) $\sin \theta = \frac{4}{5}$<br><br>$\theta_1 = 53.13^\circ$ $\theta_2 = 126.87^\circ$    | c) $\sin \theta = -0.55$<br><br>$\theta_1 = 213.37^\circ$ $\theta_2 = 326.63^\circ$                                                                                      |
| d) $\sin \theta = \frac{-\sqrt{2}}{2}$<br><br>$\theta_1 = 225^\circ$ $\theta_2 = 315^\circ$                                  | e) $\sin \theta = \frac{-\sqrt{3}}{2}$<br><br>$\theta_1 = 240^\circ$ $\theta_2 = 300^\circ$ | f) $\sin \theta = \frac{4}{\sqrt{7}}$<br><br>No solution.<br>$\frac{4}{\sqrt{7}} > 1$<br><br>$\theta_1 = \underline{\hspace{1cm}}$ $\theta_2 = \underline{\hspace{1cm}}$ |

2. Given each triangle, find the missing values and show all your work

a) Find the value of  $\angle 1$ ,  $\angle 2$ ,  $h$ ,  $BC$ , and  $AB$



$\frac{\sin 35^\circ}{15} = \frac{\sin \angle 1}{20}$   $\angle 1 = 49.9^\circ \rightarrow 130.1^\circ$   $180 - 130.1 = 49.9$   
 $\angle 2 = 49.9^\circ$   
 $\uparrow$   
has to be acute

$BC: \frac{\sin 80.2^\circ}{BC} = \frac{\sin 49.9^\circ}{15}$   $BC = 19.3$

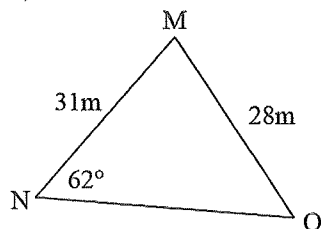
$AB: \frac{\sin 35^\circ}{15} = \frac{\sin 49.9^\circ}{AB}$   $AB = 6.7$

$h: \sin 49.9^\circ = \frac{h}{15}$   
 $h = 11.5$

$\angle 1 = 130.1^\circ$      $\angle 2 = 49.9^\circ$      $h = 11.5$      $BC = 19.3$      $AB = 6.7$



b) Find the value of  $\angle MON$ ,  $\angle OMN$ , and  $\overline{ON}$ .



$$\frac{\sin 62^\circ}{28} = \frac{\sin \angle MON}{31}$$

$$\angle MON = 77.8^\circ \text{ or } 102.2^\circ$$

$$\text{When } \angle MON = 77.8^\circ, \frac{\sin 40.2^\circ}{ON} = \frac{\sin 62^\circ}{28} \quad ON = 20.5$$

$$\text{When } \angle MON = 102.2^\circ, \frac{\sin 15.8^\circ}{ON} = \frac{\sin 62^\circ}{28} \quad ON = 8.6$$

$$\angle MON = \underline{77.8^\circ} \text{ (ACUTE)}$$

$$\angle OMN = \underline{40.2^\circ}$$

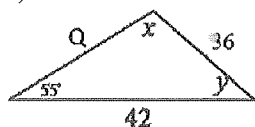
$$ON = \underline{20.5}$$

$$\angle MON = \underline{102.2^\circ} \text{ (OBTUSE)}$$

$$\angle OMN = \underline{15.8^\circ}$$

$$ON = \underline{8.6}$$

c)



$$\frac{\sin 55^\circ}{36} = \frac{\sin x}{42}$$

$$x = 72.88^\circ$$

$$y = 180 - 72.88 - 55$$

$$y = 52.12^\circ$$

$$Q = \frac{\sin 55^\circ}{36} = \frac{\sin 52.12^\circ}{Q}$$

$$Q = 34.7$$

Case #2

$$x = 107.12^\circ \quad (180 - 72.88)$$

$$y = 17.88^\circ$$

$$Q: \frac{\sin 55^\circ}{36} = \frac{\sin 17.88^\circ}{Q}$$

$$Q = 13.5$$

$$\angle x = \underline{72.88^\circ} \text{ (ACUTE)}$$

$$\angle y = \underline{52.12^\circ}$$

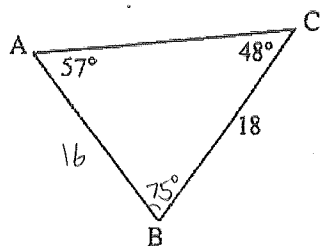
$$Q = \underline{34.7}$$

$$\angle x = \underline{107.12^\circ} \text{ (OBTUSE)}$$

$$\angle y = \underline{17.88^\circ}$$

$$Q = \underline{13.5}$$

3. Find the area of the following triangle. Note the area of a triangle is  $A = b \times h \times 0.5$ :



$$\frac{\sin 57^\circ}{18} = \frac{\sin 48^\circ}{AB}$$

$$AB = 16$$

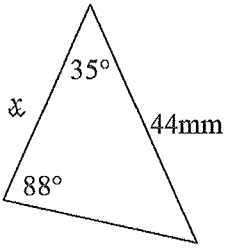
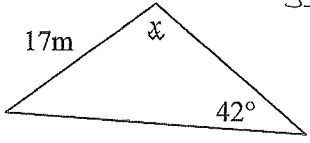
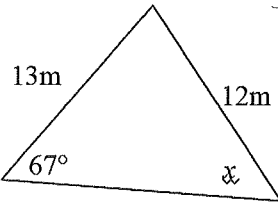
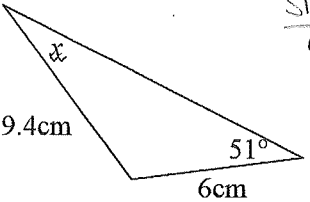
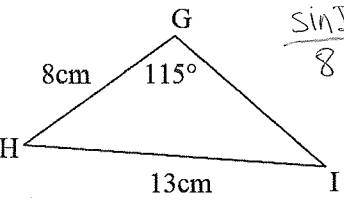
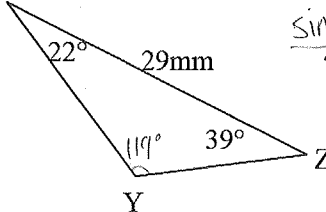
$$\frac{1}{2} ab \sin C$$

$$= \frac{1}{2} (16)(18) \sin 75^\circ$$

$$= 139 \text{ unit}^2$$



4. Given each of the following triangles, indicate whether if there would be an ambiguous case. State the reason why or why not: Solve for "x".

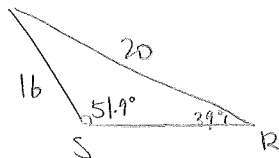
|                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                          |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>a)</p>  $\frac{\sin 88^\circ}{44} = \frac{\sin 35^\circ}{x}$ $x = 36.9 \text{ mm}$ <p>no ambiguous case</p>                                                                                      | <p>b)</p> <p>SSA. ambiguous case here</p>  $\frac{\sin 42^\circ}{17} = \frac{\sin x}{22}$ $x = 60^\circ$ $180 - 60 = 120^\circ$ $x = 120^\circ$ <p><math>x = 60^\circ \text{ or } 120^\circ</math></p> |
| <p>c)</p> <p>SSA. ambiguous.</p>  $\frac{\sin 67^\circ}{12} = \frac{\sin x}{13}$ $x = 85.72^\circ$ <p>or</p> $180 - 85.72 = 94.3^\circ$ <p><math>x = 85.72^\circ \text{ or } 94.3^\circ</math></p> | <p>d)</p>  $\frac{\sin 51^\circ}{9.4} = \frac{\sin x}{6}$ $x = 29.7^\circ$ <p>x cannot be obtuse</p> <p>no ambiguous case</p>                                                                         |
| <p>e)</p>  $\frac{\sin I}{8} = \frac{\sin 115^\circ}{13}$ $I = 33.9^\circ$ <p>no ambiguous case</p>                                                                                               | <p>f)</p> <p><math>\angle Y = 180 - 22 - 39 = 119^\circ</math></p>  $\frac{\sin 119^\circ}{29} = \frac{\sin 39^\circ}{x} = \frac{\sin 22^\circ}{y}$ $xy = 20.9 \text{ mm}$ $yz = 12.4 \text{ mm}$    |



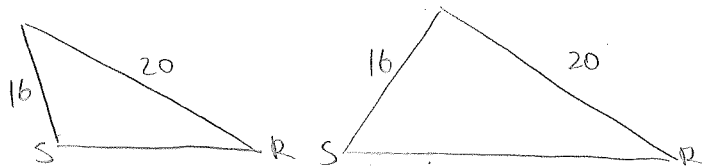
5. A lighthouse at point Q is 20 km from a yacht at point R and 16 km from a sailboat at point S. From the yacht, the lighthouse and the sailboat are separated by an angle of  $39^\circ$

- a) Is it necessary to consider the ambiguous case? Explain.

Yes.



- b) Sketch all possible diagrams for this situation.



- c) Determine all possible the distances from the yacht to the sailboat, to the nearest tenth of a kilometre.

$$S = 51.9^\circ \text{ or } 128.1^\circ$$

if  $51.9^\circ$ :

$$SR: \frac{\sin 89.1^\circ}{SR} = \frac{\sin 51.9^\circ}{20}$$

$$SR = 25.42 \text{ km}$$

if  $S = 128.1^\circ$

SR:

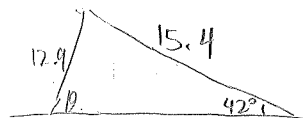
$$\frac{\sin 12.9^\circ}{SR} = \frac{\sin 128.1^\circ}{20}$$

$$SR = 5.67 \text{ km}$$

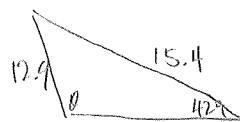
6. Jason and Sammy are part of a scientific team studying clouds. The team is about to launch a weather balloon into an active part of the cloud. Jason's rope is 15.4 m long and makes an angle of  $42^\circ$  with the ground. Belle's rope is 12.9 m long.

- a) Is it necessary to consider the ambiguous case? Explain.

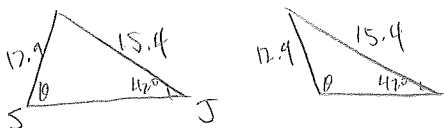
This is SSA. Yes.



$$\frac{\sin 42^\circ}{12.9} = \frac{\sin \theta}{15.4} \quad \theta = 53^\circ$$



- b) Sketch all possible diagrams for this situation.



- c) Determine all possible the distances between Jason and Sammy to the nearest tenth of a meter.

$\theta$  could be  $53^\circ$  or  $127^\circ$

if  $\theta = 53^\circ$

$$JS: \frac{\sin 53^\circ}{15.4} = \frac{\sin 85^\circ}{JS}$$

$$JS = 19.2 \text{ m}$$

if  $\theta = 127^\circ$

$$JS: \frac{\sin 127^\circ}{15.4} = \frac{\sin 11^\circ}{JS}$$

$$JS = 3.68 \text{ m}$$

